



বিদ্যাসাগর বিশ্ববিদ্যালয়

**VIDYASAGAR UNIVERSITY**

**B.Sc. Honours Examination 2021**

**(CBCS)**

**4th Semester**

**PHYSICS**

**PAPER—C8T & C8P**

**MATHEMATICAL PHYSICS III**

*Full Marks : 60*

*Time : 3 Hours*

*The figures in the right-hand margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**THEORY : C8T**

Answer any *two* questions.

2×15

1. (a) Solve the coupled ordinary linear Differential Equation

$$\dot{x} = 5x - 3y$$

$$\dot{y} = -6x + 2y$$

- (b) Locate and classify the singular point(s) and evaluate the residue(s) of  $\frac{e^{1/z}}{z^2}$ . 9+6

2. (a) Using Cayley Hamilton Theorem find the inverse matrix  $\begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$

(b) Evaluate  $\oint_C \frac{\cosh iz}{z^2 + 4z + 3} dz$

where C is the circle having  $|z| = 2$ .

- (c) Using contour integration evaluate the real integral

$$\int_0^\infty \frac{1}{1+x^2} dx = \frac{\pi}{2} \quad 5+5+5$$

3. (a) What is Cauchy Riemann condition? Apply on the function  $f(z) = |z|^2$  and comment on its analyticity.

- (b) Use residue theorem to evaluate:

$$\int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta + \sin\theta}$$

- (c) Find the Fourier transform of the function  $f(x) = e^{(-x^2)}$  5+5+5

4. (a) For Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \text{and} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Show that (i)  $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = I$  and (ii)  $[\sigma_i, \sigma_j] = 2i\sigma_k$ , where i, j, k follow cyclic order.

(b) Verify Cayley-Hamilton theorem for  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ .

(c) Find the eigen values of  $\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$ .

(d) Show that  $i(A - A^\dagger)$  is a Hermitian matrix. [3+(1+1+1)]+3+3+3

Answer any one question.

1×10

5. (a) Verify whether the following matrix is orthogonal:

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}.$$

(b) Verify whether  $\begin{pmatrix} 3/5 & 4i/5 \\ -4i/5 & -3/5 \end{pmatrix}$  is unitary.

(c) Prove that the eigen values of Hermitian matrix are real and the eigen vectors of a Hermitian matrix are orthogonal. 3+3+4

6. (a) Find Fourier Cosine transform of  $f(x) = e^{-ax}, (a > 0, x \geq 0)$

(b) Find the Taylor series expansion of a function of the complex variable

$$f(z) = \frac{1}{(z-1)(z-3)} \text{ about the point } z = 4. \quad 5+5$$

**PRACTICAL : C8P**Answer any *one* question.

1×20

1. Write a Python programme to evaluate the Fourier coefficients of the following function :

$$f(x) = \begin{cases} 0 & \text{for } -2 \leq x \leq 0 \\ 4 & \text{for } 0 \leq x \leq 2 \end{cases}$$

2. Write a Python programme script to solve the differential equation:

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

3. Write a python program to evaluate  $\frac{1}{\sqrt{2\pi\sigma^2}} \int e^{-\frac{(x-2)^2}{2\sigma^2}} (x+3) dx$

for  $\sigma = 1, 0.1$ .

**[Internal assessment - 10]**

**[Attendance - 5]**

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